

MCS 107 Final Exam

1) Evaluate the following integrals;

a) $\int x^3 \ln x dx$ b) $\int \frac{\ln^3 x}{x} dx$ c) $\int \frac{(1 + x^{\frac{1}{3}})^6}{x^{\frac{2}{3}}} dx$

d) $\int (x^2 - 5x)^9 \cdot (2x - 5) dx$ e) $\int_{-5}^5 x^{2011} \cdot (x^4 + e^{x^2} + 107) dx$

2) Two numbers have sum 16. What are the numbers if the product of the cube of one and the fifth power of the other is maximum.

3) Let $f(x) = \sqrt{3x + 2}$ be given. Using definition of the derivative, find $f'(1)$.

4) Consider the function $f(x) = \frac{x^2 + 2x + 4}{2x}$.

- Find all asymptotes
- Find local maximum and local minimum points
- Find intervals of increasing and decreasing
- Find intervals of concave up and concave down
- Sketch the graph

5) Find the area between the curves $y = 2 - 2(x - 1)^2$ and $y = -x$ over the interval $[0, 1]$.

6) Find the equation of the tangent line to the curve $y = 3^x + x^3 + 1$ at point $x = 0$.

7) If the marginal cost for a company is $C'(x) = x^2 \cdot 2^{x^3+1}$ and the cost for producing 2 units is $\frac{254}{\ln 2}$, that is $C(2) = \frac{254}{\ln 2}$. Find $C(x)$.