ÇANKAYA UNIVERSITY

Department of Mathematics

MATH 107 - Calculus for Business and Economics I

FINAL EXAMINATION
22.08.2017

1) Evaluate the following limits if it exists:

a) $\lim_{x \to -2} \frac{2x^2+5x+2}{x^2-5x-14} \underset{L.R.}{=} \lim_{x \to -2} \frac{4x+5}{2x-5} = \frac{1}{3}$

b) $\lim_{x \to \infty} \frac{(3x+1)(5x^2+3)}{x\,(1-2x-5x^2)} = \lim_{x \to \infty} \frac{15x^3}{-5x^3} = -3$

c) $\lim_{x \to 1} \frac{x - \ln x - 1}{x^2-1} \underset{L.R.}{=} \lim_{x \to 1} \frac{1-\frac{1}{x}}{2x} = 0$

2) Find $f'(x)$ if

a) $f(x) = (e^{x^2+2}+2)^7$

$f'(x) = 7(e^{x^2+2}+2)^6\, e^{x^2+2}\, 2x$

b) $f(x) = \frac{x^2+1}{2x-3} + (x-5)^2$

$f'(x) = \frac{2x^2-6x-2}{(2x-3)^2} + 2(x-5)$

c) $f(x) = (x+1)^3 \ln(x+1)$

$f'(x) = 3(x+1)^2 \ln(x+1) + (x+1)^3 \frac{1}{x+1} = (x+1)^2(3\ln(x+1)+1)$

3) Evaluate the following integrals;

a) $\displaystyle\int\left(2x^5 - x\sqrt{x} + \frac{1}{x^2} - 5\right)dx = \frac{x^6}{3} - \frac{2x^{5/2}}{5} - \frac{1}{x} - 5x + \text{C}$

b) $\displaystyle\int_1^2 (4x-3)(x^2-2)\,dx = \int_1^2 (4x^3 - 3x^2 - 8x + 6)dx = x^4 - x^3 - 4x^2 + 6x\Big|_1^2 = 2$

c) $\displaystyle\int \frac{(x+1)^2}{x}dx = \int \frac{x^2+2x+1}{x}dx = \int\left(x + 2 + \frac{1}{x}\right)dx = \frac{x^2}{2} + 2x + \ln x + \text{C}$

d) $\displaystyle\int \frac{2x+3}{(x^2+3x-4)^5}dx = \int \frac{du}{u^5} = \frac{u^{-4}}{-4} + \text{C} = \frac{-(x^2+3x-4)^{-4}}{4} + \text{C}$

$u = x^2 + 3x - 4 \;\Rightarrow\; du = (2x+3)dx$

4) Evaluate the following integrals;

a) $\displaystyle\int e^{x^4+x^2}(2x^3+x)dx = \int e^u \frac{du}{2} = \frac{e^u}{2} + \text{C} = \frac{e^{x^4+x^2}}{2} + \text{C}$

$u = x^4 + x^2 \;\Rightarrow\; du = (4x^3 + 2x)dx$

b) $\displaystyle\int \frac{x^2-3}{x^3-9x+5}dx = \int \frac{du}{3u} = \frac{\ln u}{3} + \text{C} = \frac{\ln(x^3-9x+5)}{3} + \text{C}$

$u = x^3 - 9x + 5 \;\Rightarrow\; du = (3x^2 - 9)dx$

c) $\displaystyle\int x^9 \ln x\,dx = \frac{x^{10}\ln x}{10} - \int \frac{x^{10}}{10x}dx = \frac{x^{10}\ln x}{10} - \frac{x^{10}}{100} + \text{C}$

$u = \ln x \;\Rightarrow\; du = \dfrac{1}{x}dx$

$dv = x^9 dx \;\Rightarrow\; v = \dfrac{x^{10}}{10}$

d) $\displaystyle\int \frac{2x-2}{x^2-2x-8}dx = \int\left(\frac{1}{x-4} + \frac{1}{x+2}\right)dx = \ln(x-4) + \ln(x+2) + \text{C}$

$\dfrac{2x-2}{x^2-2x-8} = \dfrac{A}{x-4} + \dfrac{B}{x+2} \;\Rightarrow\; \dfrac{2x-2}{x^2-2x-8} = \dfrac{1}{x-4} + \dfrac{1}{x+2}$

5) a) If y is a function of x such that $\dfrac{dy}{dx} = 2x + \dfrac{1}{\sqrt{x}}$ and $y(1) = 4$, find $y(x)$.

$y(x) = x^2 + 2\sqrt{x} + C$ and $y(1) = 4 \Rightarrow$ C$=1$

Then $y(x) = x^2 + 2\sqrt{x} + 1$

b) Find the area of the region between the curves $y = x^2 + 1$ and $y = 2x + 4$.

These curves intersect at the points $x = -1$ and $x = 3$. Then

Area$= \displaystyle\int_{-1}^{3} [(2x-4) - (x^2+1)]dx = \int_{-1}^{3} (-x^2 + 2x + 3)dx = \frac{-x^3}{3} + x^2 + 3x\Big|_{-1}^{3} = \frac{32}{3}$