ÇANKAYA UNIVERSITY

Department of Mathematics

MCS 107 - Calculus for Business and Economics I

SECOND MIDTERM EXAMINATION
08.08.2016

1) Find $f'(x)$ if

a) $f(x) = \dfrac{2x^5 + 5x^2 + 2\sqrt{x} - 1}{x}$.

$f'(x) = 8x^3 + 5 - x^{-3/2} + x^{-2}$

b) $f(x) = (x^2 - 5)(3x^4 - 2x^2)$

$f'(x) = 2x(3x^4 - 2x^2) + (x^2 - 5)(12x^3 - 4x)$

c) $f(x) = (3x^3 - 5x)^4$

$f'(x) = 4(3x^3 - 5x)^3(9x^2 - 5)$

d) $f(x) = \dfrac{x^2 - 3x}{2x + 1}$

$f'(x) = \dfrac{2x^2 + 2x - 3}{(2x + 1)^2}$

2) Find $f'(x)$ if

a) $f(x) = e^{2x^4+5x^2}\sqrt{x}$.

$$f'(x) = e^{2x^4+5x^2}(8x^3 + 10x)\sqrt{x} + \frac{1}{2}e^{2x^4+5x^2}x^{-1/2}$$

b) $f(x) = \ln(7x^3 - 2x)$

$$f'(x) = \frac{21x^2 - 2}{7x^3 - 2x}$$

c) $f(x) = 4^x\sqrt{x^2 - 1}$

$$f'(x) = 4^x \ \ln 4 \, (x^2 - 1)^{1/2} + 4^x \, x \, (x^2 - 1)^{-1/2}$$

d) $f(x) = \dfrac{\ln x}{x^2}$

$$f'(x) = \frac{1 - 2\ln x}{x^3}$$

3) a) Find the equation of the tangent line to the graph of $f(x) = x^2 - \dfrac{1}{x-1}$ at the point $(2, 3)$.

Solution:

$f'(x) = 2x + \dfrac{1}{(x-1)^2}$

$m_{\tan} = f'(2) = 5$

Equation of tangent line: $y = 5x - 7$

b) If $x^3y^2 - 4y^2 + 3x^3 = 2y - x - 12$ is given, evaluate $y'(x)$ at the point $(1, 2)$.

Solution:

Differentiate both parts of the equation:

$3x^2y^2 + 2x^3yy' - 8yy' + 9x^2 = 2y' - 1$

At the point $(1, 2)$: $y' = \dfrac{11}{7}$

4) Draw a graph of $f(x) = 2x^3 - 6x^2 - 18x + 2$ showing all significant features. That is,

a) Find f' and f''.
b) Draw a table showing the signs of f' and f'' and find local extrema and inflection points. (If any)
c) Sketch the graph.

Solution:

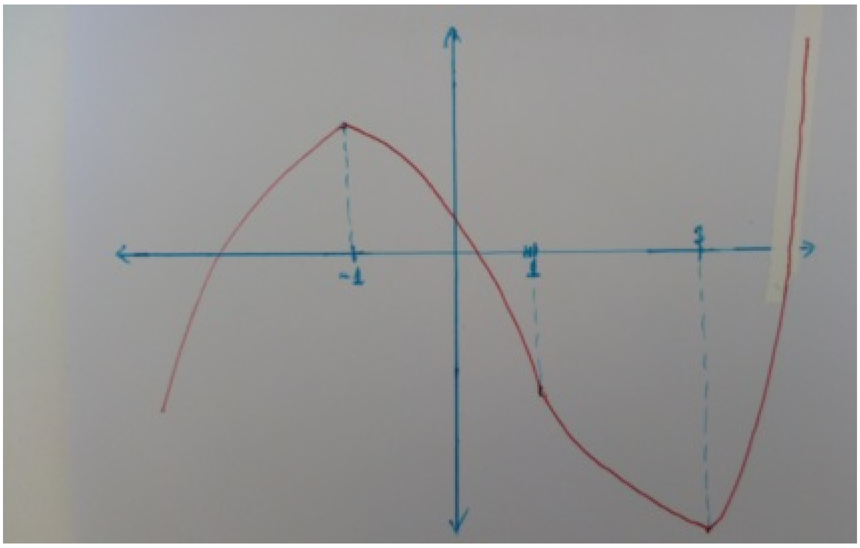
$f'(x) = 6x^2 - 12x - 18$

$f''(x) = 12x - 12$

	-1 1 3			
f'	+	-	-	+
f''	-	-	+	+

f has an local max. at $x = -1$, local min. at $x = 3$ and inflection point at $x = 1$.

y-int.:$(0, 2)$



5) a) The total revenue from the sales of a certain product are given by

$$R(q) = 1 + \frac{q^2 + 50}{q} + (q - 49)^3.$$

Find the marginal revenue when $q = 50$.

Solution:

Marginal revenue= $R'(q) = 1 - \dfrac{50}{q^2} + 3(q - 49)^2$

At $q = 50$: $\quad R'(50) = \dfrac{199}{50}$

b) Find two numbers such that $2x + y = 8$ and $\quad x^2 - y \quad$ is a minimum.

Solution:

$y = 8 - 2x \Rightarrow f(x) = x^2 - (8 - 2x) = x^2 + 2x - 8$

$f'(x) = 2x + 2 = 0 \Rightarrow x = -1$

Since $f''(x) = 2 > 0$, $x = -1$ is a minimum.