ÇANKAYA UNIVERSITY

Department of Mathematics

MCS 107 - Calculus for Business and Economics I

FIRST MIDTERM EXAMINATION
24.07.2017

Question	Grade	Out of
1		20
2		18
3		15
4		14
5		15
6		20
Total		102

STUDENT NUMBER:

NAME-SURNAME:

SIGNATURE:

INSTRUCTOR:

DURATION: 80 minutes

IMPORTANT NOTES:

1) Please make sure that you have written your student number and name above.

2) Check that the exam paper contains 6 problems.

3) Show all your work. No points will be given to correct answers without reasonable work.

1) Find the solution sets of the following expressions.

a) $\dfrac{8}{x^2 - x - 6} = \dfrac{2}{x - 3} + \dfrac{1}{x + 2}$ $\implies$ $x = \dfrac{7}{3}$

b) $(x - 2)^2 + 5x - 6 = 0$ $\implies$ $x^2 + x - 2 = 0$ $\implies$ $x = 1,\ x = -2$

c) $\sqrt{2x + 7} = x - 4$ $\implies 2x + 7 = x^2 - 8x + 16$ $\implies x^2 - 10x + 9 = 0$ $\implies$ $x = 1, x = 9$

For $x = 1$ $\sqrt{9} = -3$ but it is not possible so only solution is $x = 9$.

d) $\left|\dfrac{3x - 1}{2}\right| < 4$ $\implies$ $-4 < \dfrac{3x - 1}{2} < 4$ $\implies$ $\dfrac{-7}{3} < x < 3$ $\implies$ $x \in \left(\dfrac{-7}{3}, 3\right)$

2) Let $f(x) = x^2 - 1$ and $g(x) = \dfrac{1}{x + 1}$.

a) Find $(fog)(x)$, $(gof)(x)$, $(f - g)(x)$ and $(fg)(x)$

b) Evaluate $(f + 3g)(0)$ and $(fg)(2)$.

- $(fog)(x) = \left(\dfrac{1}{x + 1}\right)^2 - 1$

- $(gof)(x) = \dfrac{1}{x^2}$

- $(f - g)(x) = x^2 - 1 - \dfrac{1}{x + 1} = \dfrac{x^3 + x^2 - x - 2}{x + 1}$

- $(fg)(x) = x - 1$

- $(f + 3g)(0) = 2$

- $(fg)(2) = 1$

3) For the function $f(x) = x^2 - 4x - 12$,

a) Find vertex, x-intercept and y-intercept points.
b) Find Domain(f) and Range(f).
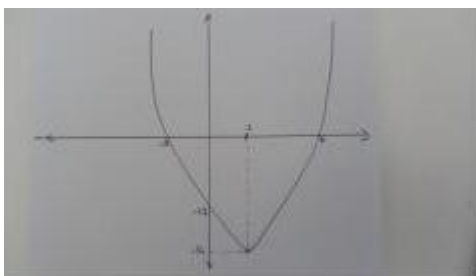**c)** Sketch the graph of the function.

Vertex: $(2, -16)$
y-intercept: $(0, -12)$
x-intercepts: $(-2, 0)$, $(6, 0)$
Domain(f): $(-\infty, \infty)$
Range(f): $[-16, \infty)$

4) a) Find equation of a line passing through the point $(1, -2)$ and perpendicular to the line $2x + y + 3 = 0$.

Slope of the line $2x + y + 3 = 0$ $m_1 = -2$
Since lines are perpendicular $mm_1 = -1$ $\implies$ $m = 1/2$
Line equation: $y - y_1 = m(x - x_1)$ $\implies$ $2y - x + 5 = 0$

b) Find the compound amount if 1000 TL is invested for 2 years at 8% compounded quarterly.

$$S = P(1 + r)^n = 1000(1 + 0.02)^8 = 1000(1.02)^8$$

5) Solve the following equalities.

a) $e^{x+1} - 1 = 2$

$e^{x+1} = 3$ $\implies$ $x + 1 = \ln 3$ $\implies$ $x = \ln 3 - 1$

b) $\log_5 50 - \log_5 2 = \log_5 100 - x$

$$x = \log_5 100 - \log_5 50 + \log_5 2 = \log_5 \left(\frac{100 \cdot 2}{50} \right) = \log_5 4$$

c) $\ln(x + 6) - \ln(2) = 2 \ln x$

$\ln \left(\dfrac{x+6}{2} \right) = \ln(x^2)$ $\implies$ $\dfrac{x+6}{2} = x^2$ $\implies$ $2x^2 - x - 6 = 0$ $\implies$
$x = 2, x = -3/2$ but $\ln(-3/2)$ is undefined only solution is $x = 2$.

6) a) Evaluate the following limits.

i) $\displaystyle \lim_{x \to 5} \frac{x - 5}{\sqrt{x - 1} - 2} = \lim_{x \to 5} \frac{(x - 5)(\sqrt{x - 1} + 2)}{x - 5} = 4$

ii) $\displaystyle \lim_{x \to \infty} \frac{x^2 - 1}{4 - 2x^2} = \lim_{x \to \infty} \frac{x^2}{-2x^2} = \frac{-1}{2}$

iii) $\displaystyle \lim_{x \to 3} \frac{x - 3}{x^2 - 2x - 3} = \lim_{x \to 3} \frac{x - 3}{(x - 3)(x + 1)} = \frac{1}{x + 1} = \frac{1}{4}$

b) Let $f(x) = \begin{cases} \dfrac{x}{x^2 + x + 1}, & \text{if } x < 0 \\ 1, & \text{if } x = 0 \\ e^{x^2 + 2x} - 1, & \text{if } x > 0. \end{cases}$

Is $f(x)$ continuous at $x = 0$? Verify your answer.

$\displaystyle \lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x) = 0$ $\implies$ $\displaystyle \lim_{x \to 0} f(x) = 0$ but since $f(0) = 1 \neq \lim_{x \to 0} f(x)$, f is not continuous.